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Gravitational GUT Breaking and the GUT-Planck Hierarchy

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Abstract

It is shown that non-renormalizable gravitational interactions in the Higgs sector of supersymmetric grand unified theories (GUT's) can produce the breaking of the unifying gauge group G at the GUT scale $M_{\rm GUT} \sim 10^{16}$ GeV. Such a breaking offers an attractive alternative to the traditional method where the superheavy GUT scale mass parameters are added ad hoc into the theory. The mechanism also offers a natural explanation for the closeness of the GUT breaking scale to the Planck scale. A study of the minimal SU(5) model endowed with this mechanism is presented and shown to be phenomenologically viable. A second model is examined where the Higgs doublets are kept naturally light as Goldstone modes. This latter model also achieves breaking of G at $M_{\rm GUT}$ but cannot easily satisfy the current experimental proton decay bound.

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The precision LEP data has indicated a unification, with the help of supersymmetry, of the three coupling constants of the Standard Model at a scale of $M_{\rm GUT} \sim 10^{16}$ GeV, thus indicating the existence of a hierarchy between the Planck scale and the grand unification scale. This hierarchy has allowed model builders to build various GUTs as effective field theories, sometimes including non-renormalizable operator (NRO) terms scaled by inverse powers of the Planck mass to account for gravitational effects. Such models with additional NRO terms can successfully account for lower values of $\alpha_3(M_Z)$ [4,5] should the lower energy measurements of that parameter turn out to be correct. They can also affect various low energy predictions such as that of $\tan\beta$ from the m_b/m_τ constraint [6]. In these field theoretic approaches, the GUT gauge group is broken and the hierarchy is achieved by the ad hoc inclusion of $O(M_{\rm GUT})$ mass terms.

The smallness of the GUT-Planck hierarchy, on the other hand, suggests that the true GUT may actually reside at the Planck scale, unified with gravity. However, one would then expect all mass parameters to be $O(M_{\rm Pl})$, i.e. a particle would either be massless or a member of a tower of Planck mass states. The inclusion of $O(M_{\rm GUT})$ mass terms would then be difficult to justify. In this letter, we point out that without any $O(M_{\rm GUT})$ mass parameters, one may still achieve gauge group breaking via the NRO terms. In addition, the spectrum of masses produced in this type of GUT breaking is generally below the Planck scale, so that the coupling constant unification naturally occurs at O(10^{16} GeV), despite the absence of mass parameters of such size. The specific spectrum and its parametrization is model dependent, however, and we will examine below two models and their phenomenological viabilities.

Within the context of string theory, gauge coupling unification below the string scale of $M_{\rm string} \sim g_{string} \times 5 \times 10^{17}$ GeV has been problematic [7]. The models we consider make use of adjoint representations to break the gauge group. We note that SU(5) and SO(10) GUTs from Kac-Moody (KM) level 2 strings allow the existence of such adjoint representations with the requirement that there be no mass terms in the superpotential [8]. Furthermore, though as yet not realistic, an explicit three generation SU(5) example of a KM level 2 model

has now been constructed [9]. Although we do not consider here string models explicitly, our work suggests the possible realization of such a model.

Our first model is a simple modification of the minimal SU(5) SUSY-GUT where the mass term for the adjoint Higgs has been eliminated, and instead the leading NRO self interaction terms have been added. The superpotential of the symmetry breaking sector of our SU(5) model, up to $O(1/M_{\rm Pl})$ [10], is

$$W = \frac{1}{3}\lambda_0 {\rm tr}\Sigma^3 + \frac{1}{4}\frac{\lambda_1}{M_{\rm Pl}}({\rm tr}\Sigma^2)^2 + \frac{1}{4}\frac{\lambda_2}{M_{\rm Pl}}{\rm tr}\Sigma^4 + W_H \tag{1}$$

where Σ is a $\mathbf{24}$ of SU(5), W_H couples Σ to the $\mathbf{5}$ and $\overline{\mathbf{5}}$ Higgs, and $M_{\rm Pl} = 1/\sqrt{8\pi G_N} \simeq 2.4 \times 10^{18}$ GeV. We might imagine the $\lambda_{1,2}$ terms to arise from integrating out Planckian mass states. The effective potential for scalar fields is given by $V = \sum_i |\frac{\partial W}{\partial \phi_i}|^2$, where ϕ_i is the scalar component of the multiplet. Minimizing the effective potential, one finds the VEV of Σ that breaks the gauge group into SU(3) $\times$ SU(2) $\times$ U(1),

$$\langle\Sigma\rangle = {\rm diag}(2, 2, 2, -3, -3)\Sigma_0, \tag{2}$$

where

$$\Sigma_0 = \frac{\lambda_0}{(30\lambda_1 + 7\lambda_2)} M_{\rm Pl}. \tag{3}$$

Here, Σ_0 plays the role of $M_{\rm GUT}$. It is the smallness of this VEV for reasonable values of $\lambda_{0,1,2}$ which leads to the GUT-Planck hierarchy in this model. Expanding Σ around this VEV, $\Sigma = \langle\Sigma\rangle + \Sigma'$, one finds that the $(\mathbf{3}_{SU(3)}, \mathbf{2}_{SU(2)}, (5/3)_{U(1)})$ components become Goldstone bosons giving rise to a mass for the super-heavy vector bosons, $M_V = 5\sqrt{2} g_5 \Sigma_0$, where g_5 is the GUT gauge coupling constant. Note that the large factor of 30 in the denominator of Eq. (3), which is instrumental in achieving the GUT-Planck hierarchy, is purely group theoretical in origin, coming from $\langle {\rm tr}\Sigma^2\rangle$. The remaining components of Σ' , ((**8**,**1**,0), (**1**,**3**,0), and (**1**,**1**,0)), grow masses:

$$M_{\Sigma_8} = 5\lambda_0 \frac{15\lambda_1 + 4\lambda_2}{30\lambda_1 + 7\lambda_2}\Sigma_0,$$

$$
\begin{aligned}
M_{\Sigma_3} &= \frac{15}{2}\lambda_0 \frac{10\lambda_1 + \lambda_2}{30\lambda_1 + 7\lambda_2}\Sigma_0, \\
M_{\Sigma_1} &= \frac{1}{2}\lambda_0\Sigma_0.
\end{aligned} \tag{4}
$$

For W_H we consider the simplest choice,

$$
W_H = \lambda' H(\Sigma + 3M')\overline{H}, \tag{5}
$$

where H and $\overline{H}$ are the Higgs $\mathbf{5}$ and $\overline{\mathbf{5}}$. Here, the mass parameter M' is fine tuned to equal Σ_0 so as to generate a pair of massless SU(2) doublets which break the electro-weak symmetry [11]. We look later at more natural solutions to this well known fine tuning problem. The remaining components of H and $\overline{H}$, ($\mathbf{3}$,$\mathbf{1}$,2/3), then grows a mass

$$
M_{H_3} = 5\lambda'\Sigma_0. \tag{6}
$$

Defining M_U as the largest of these masses, we see that it is easy to get $M_U \sim 10^{16}$ GeV, for example with $\lambda_0 = 0.1$, $\lambda_1 = 1$, $\lambda_2 = 1$, $\lambda' = 1$ and $g_5 = 0.7$ one finds $M_U = M_{H_3} = 3.24 \times 10^{16}$ GeV. Note that this model has the same particle content as the minimal SU(5) model except that the octet and the triplet of Σ are not degenerate when $\lambda_2 \neq 0$. In general this non-degeneracy affects coupling constant unification [12] and can change the prediction of α_3, although not as strongly as the NRO which we discuss next.

To investigate gauge coupling unification, we add to our Lagrangian a term $(c/2M_{\rm Pl})\mathrm{tr}(\Sigma F F)$ [13,14], which has a significant effect on the matching conditions at the high scale [4,5]. This term can arise naturally in supergravity from the expansion of the gauge kinetic energy function $f_{\alpha\beta} = \delta_{\alpha\beta} + \frac{1}{M_{\rm Pl}} a^i_{\alpha\beta\gamma}\phi^\gamma_i + \cdots$ for $\phi^\gamma_i = \Sigma$. (This term, when ϕ^γ_i is the Polonyi field that spontaneously breaks supersymmetry, similarly gives rise to the gaugino soft breaking mass, so it is not unnatural to also have a term where $\phi^\gamma_i = \Sigma$.) A Monte Carlo exploration has been done using the "naturalness" condition that $|c|$ and each $|\lambda|$ lie between 0.1 and 2. For each point in this parameter space, values of $\alpha_3(M_Z)$, $\sin^2(\theta_W)$, and g_5 were determined by requiring that the coupling constants unify at M_U. We run the coupling constants using the 2-loop renormalization group equations (RGE's)

in the $\overline{\mathrm{MS}}$ scheme including the particle content of the MSSM and the SU(5) GUT as described above [15]. Each point is then checked to see if it satisfies the proton decay bound, $\tau(p \to \bar{\nu}K) > 1 \times 10^{32}$ yr (90% CL) [16] which leads to $M_{H_3} > 1.2 \times 10^{16}$ GeV [17]. The results appear as Fig. 1 [18]. We find that phenomenologically acceptable values of $\alpha_3(M_Z)$ and $\sin^2(\theta_W)$ generally require λ_0 to be small, $\lesssim 0.3$, while $\lambda_1 \gtrsim 0.5$. The proton decay constraint does restrict the parameter space but only marginally, requiring $\lambda' \gtrsim 0.2$.

We turn now to a model that avoids the doublet-triplet splitting problem. There are several known ways to naturally make the Higgs doublets massless. Of these, an attractive method is to assume that there is a global symmetry in the Higgs sector of the GUT which when broken (effected by the breaking of the unifying gauge group) results in the Higgs doublets becoming the Goldstone modes, and thus massless [19–23]. An elegant realization of this idea is to embed the local SU(5) symmetry into a larger global SU(6). We now examine this GUT modified in a minimal way, in the same manner as our first model, so that the NRO's are given the role to break the unifying gauge symmetry. We consider a superpotential of the form

$$W = \frac{1}{3}\lambda_0 \mathrm{tr}\Sigma^3 + \frac{\lambda_1}{4M_{\mathrm{Pl}}}(\mathrm{tr}\Sigma^2)^2 + \frac{\lambda_2}{4M_{\mathrm{Pl}}}\mathrm{tr}\Sigma^4, \tag{7}$$

where now Σ is a $\mathbf{35}$ of SU(6) which decomposes to $\mathbf{24} + \mathbf{5} + \overline{\mathbf{5}} + \mathbf{1}$ under SU(5), and then to $(\mathbf{8}, \mathbf{1}, 0) + (\mathbf{1}, \mathbf{3}, 0) + (\mathbf{3}, \mathbf{2}, 5/3)_g + (\overline{\mathbf{3}}, \mathbf{2}, -5/3)_g + (\mathbf{1}, \mathbf{1}, 0) + (\mathbf{3}, \mathbf{1}, 2/3) + (\overline{\mathbf{3}}, \mathbf{1}, -2/3) + (\mathbf{1}, \mathbf{2}, 1)_g + (\mathbf{1}, \mathbf{2}, -1)_g + (\mathbf{1}, \mathbf{1}, 0)$ under SU(3) $\times$ SU(2) $\times$ U(1), where the Goldstone modes are indicated with a subscript g. The effective potential from Eq. (7) then yields the VEV which breaks the SU(6) symmetry down to SU(4) $\times$ SU(2) $\times$ U(1) (and consequently the SU(5) down to the standard model):

$$\langle\Sigma\rangle = \mathrm{diag}(1, 1, 1, 1, -2, -2)\Sigma_0, \tag{8}$$

where

$$\Sigma_0 = \frac{\lambda_0}{3(4\lambda_1 + \lambda_2)} M_{\mathrm{Pl}}. \tag{9}$$

The color triplet Goldstone modes give mass to the super-heavy vector bosons via the Higgs mechanism while the SU(2) doublets automatically remain massless and are identified as the light Higgs doublets. Thus the super-heavy spectrum is:

$$\begin{aligned} M_{\Sigma_8} = M_{H_3} = M_{\Sigma_1} &= \frac{3}{2}\lambda_0\Sigma_0, \\ M_{\Sigma_3} &= 6\lambda_0\frac{\lambda_1}{4\lambda_1+\lambda_2}\Sigma_0, \\ M_{\Sigma_1'} &= \frac{1}{2}\lambda_0\Sigma_0, \\ M_V &= 3\sqrt{2}g_5\Sigma_0. \end{aligned} \tag{10}$$

Monte Carlo investigation reveals that again, a sufficient GUT-Planck hierarchy is naturally generated and coupling unification is achieved with the current measurements of the coupling constants. However, we find that this time, the values of M_{H_3} in the region where the coupling unification is achieved is below the proton decay bound of 1.2×10^{16} GeV. The discrepancy arises due to the fact that there commonly exists a splitting between $M_{\Sigma_{3,8}}$ and M_V in these gravitationally induced GUT breaking models. In particular, for this global SU(6) model, the ratio $M_{\Sigma_8}/M_V = \lambda_0/2\sqrt{2}g_5$ must be $\lesssim 0.15$ since λ_0 needs to be $\lesssim 0.3$ in order to generate sufficient GUT-Planck hierarchy, and $g_5 \sim 0.7$. On the other hand, M_V and $M_{\Sigma_{3,8}}$ are related by $M_V^{2/3}M_{\Sigma_8}^{1/6}M_{\Sigma_3}^{1/6} = 2.0 \times 10^{16}$ GeV [24] which becomes a relation between M_{H_3} and M_V in this global SU(6) model where M_{Σ_8} is degenerate with M_{H_3}. The requirement for the splitting between M_{H_3} and M_V then forces M_{H_3} below the proton decay bound (unless $M_{\Sigma_3}/M_{\Sigma_8}$ is pushed to be unnaturally small).

It is possible to overcome this difficulty by going outside of what we above defined as "natural" values of the parameters. One possibility is to allow values of $\lambda_{1,2}$ that are above the upper bound of 2.0 which assure validity of the perturbation theory. Such large values will help in creating sufficient GUT-Planck hierarchy so that the above constraint on λ_0 and thus M_{Σ_8}/M_V is loosened. For example, $\lambda_0 = 0.627$, $\lambda_1 = 8.33$, $\lambda_2 = 6.03$, $g_5 = 0.735$, and $c = 1.4$ give $M_\Sigma = 1.02 \times 10^{16}$ GeV, $M_V = 3.98 \times 10^{16}$ GeV, $M_{H_3} = 1.20 \times 10^{16}$ GeV, $\alpha_3(M_Z) = 0.125$, and $\sin^2(\theta_W) = 0.2307$. Another possibility is to allow the ratio $M_{\Sigma_3}/M_{\Sigma_8}$ to be unnaturally small, which translates to allowing small values for λ_1/λ_2. For example,

$\lambda_0 = 0.136$, $\lambda_1 = 0.00205$, $\lambda_2 = 1.84$, $g_5 = 0.759$, and $c = 1.38$ give $M_\Sigma = 5.34 \times 10^{16}$ GeV, $M_V = 1.9 \times 10^{17}$ GeV, $M_{H_3} = 1.2 \times 10^{16}$ GeV, $\alpha_3(M_Z) = 0.125$, and $\sin^2(\theta_W) = 0.2307$. Yet another possibility is to arrange a highly non-degenerate SUSY spectrum so that their low energy threshold contribution (thus far assumed negligible) can favorably affect the predicted values of $\alpha_3(M_Z)$ and $\sin^2(\theta_W)$. We find that in general, one then needs to have a large ratio between the slepton masses and the gluino mass. If we assume gluino mass of O(100 GeV), one would then need the slepton masses to be of O(5 TeV).

In conclusion, we have shown that GUT scale can be generated naturally from M_{Pl} when the GUT symmetry is broken by NRO interactions without having to put in the GUT scale mass parameter by hand. We have explicitly demonstrated this mechanism within the context of minimal SU(5) GUT. The resulting GUT is shown to be phenomenologically viable. The global SU(6) model, which gives a natural doublet-triplet Higgs mass splitting, also generates a GUT mass scale in this fashion. However, such models have difficulty in satisfying the current proton decay bound. Nevertheless, the mechanism is quite general and applicable to many other GUTs.

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[10] We have also examined the case where $O(1/{M_{\rm Pl}}^2)$ terms have been added to the superpotential, and have found that they make only a small perturbation to our results.

[11] It is possible to achieve doublet-triplet splitting with an NRO term instead of the M' mass term by replacing $3\lambda' M'$ with $\lambda'_1 \Sigma^2/M_{\rm Pl} + \lambda'_2 {\rm tr}\Sigma^2/M_{\rm Pl}$. The fine tuning then takes the form $\lambda'/(3\lambda'_1 + 10\lambda'_2) = \lambda_0/(30\lambda_1 + 7\lambda_2)$.

[12] The effect of superheavy thresholds to α_3 can be summarized by considering the combination $2\alpha_3^{-1} - 3\alpha_2^{-1} + \alpha_1^{-1}$ of coupling constants at M_Z:

$$\alpha_3^{-1}(M_Z) = \frac{1}{2}[3\alpha_2^{-1} - \alpha_1^{-1}] - \frac{3}{5\pi}\ln\frac{M_{H_3}}{M_Z} + \frac{3}{2\pi}\ln\frac{M_{\Sigma_3}}{M_{\Sigma_8}}.$$

The last term, for our SU(5) model is $(3/2\pi)\ln((30\lambda_1 + 3\lambda_2)/(30\lambda_1 + 8\lambda_2))$. By taking the case where signs of λ_1 and λ_2 are different, this contribution can lower the acceptable range of α_3, though its effect is in general small compared to that of the

NRO $(c/2M_{\rm Pl}){\rm tr}(\Sigma FF)$, mentioned later in the main text. In our numerical search, summarized in Fig. 1, we find that the lower bound for α_3 is $\simeq 0.105$.

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[18] A color version of this plot which indicates the values of the Higgs mass as well as other relevant plots created during the course of this work are available at our Web page, http://physics.tamu.edu/~urano/research/gutplanck.html.

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[24] The combination $-5\alpha_1^{-1}+3\alpha_2^{-1}+2\alpha_3^{-1}$ leads to a one loop relation between $M_{\Sigma_{3,8}}$, M_V, and the low energy data,

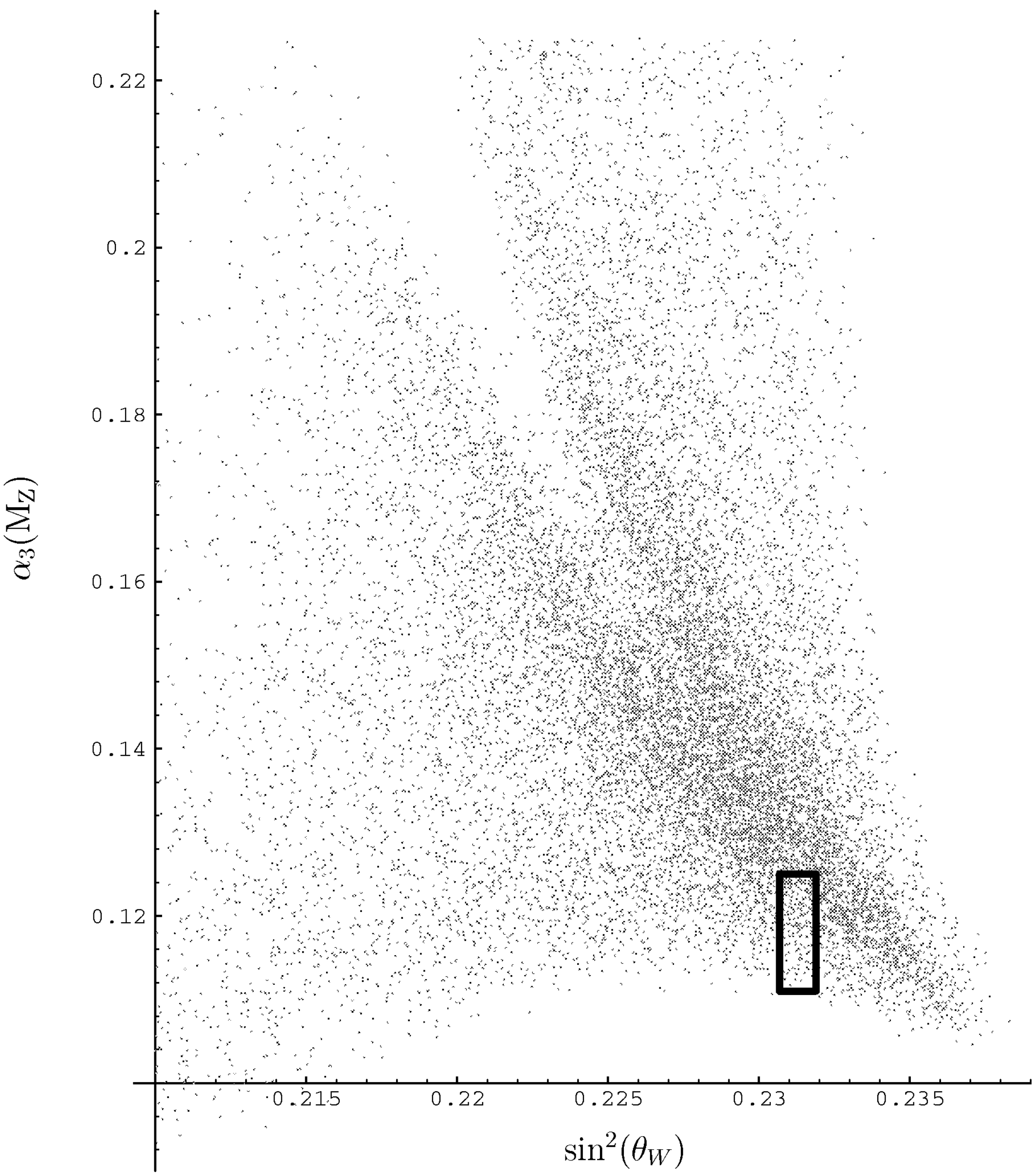
$$\begin{aligned}&\frac{1}{12\pi}\ln(\frac{M_{\Sigma_3}}{M_Z})+\frac{1}{12\pi}\ln(\frac{M_{\Sigma_8}}{M_Z})+\frac{2}{6\pi}\ln(\frac{M_V}{M_Z})\\&=\frac{1}{12}\frac{\cos(2\theta_W)}{\alpha(M_Z)}-\frac{1}{18}\frac{1}{\alpha_3(M_Z)}-\frac{2}{18\pi}\ln(\frac{M_{\rm SUSY}}{M_Z}).\end{aligned}$$

This expression is unaffected by the NRO, $(c/2M_{\rm Pl}){\rm tr}(\Sigma FF)$, and thus provides a strong relationship amongst the proton decay modes for the global SU(6) type GUTs where

$M_{H_3} = M_{\Sigma_8}$. The value in the main text is gotten from the above equation with $\sin^2(\theta_W) = 0.23129$, $\alpha_3(M_Z) = 0.118$, and $M_{\rm SUSY} = M_Z$.

FIGURES

FIG. 1. The subregion of parameter space satisfying the naturalness constraints, $0.1 < |\lambda|, |c| < 2$, and the proton decay constraint, 2.0×10^{17} GeV $> M_{H_3} > 1.2 \times 10^{16}$ GeV, projected onto the $\sin^2(\theta_W)$-$\alpha_3(M_Z)$ plane. The box corresponds to the experimentally measured value of $\sin^2(\theta_W) = 0.23129 \pm 0.00060$ and $\alpha_3(M_Z) = 0.118 \pm 0.007$. The $\sin^2(\theta_W)$ width corresponds to a 2 s.d. range around the current world average (M. Woods, talk at International Europhysics Conference on High Energy Physics, Brussels, 1995.) The upper bound on M_{H_3} represent a reasonable upper bound on the validity of a GUT field theory (above which Planck physics, e.g. strings, effects would become large).



0.22
0.2
0.18
0.16
0.14
0.12
$\alpha_3(\mathrm{M_Z})$
0.215
0.22
0.225
0.23
0.235
$\sin^2(\theta_W)$